A Critical Assessment of Gödelian Realism

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# 1 Introduction

Although unappreciated in his own day, Kurt Gödel is now respected both for his robust defense of Platonic realism and for the serious challenge that he posed to many of the positivistic philosophies of his own day. But while Gödel is perhaps more now than ever appreciated for his historical contributions to the philosophy of mathematics, his realism is too often relegated passed over as a point of dialogue in favor of the seemingly more “mature” realism of philosophers such as Quine and Putnam.

In light of this situation, the present essay seeks to bring forth a fuller assessment of Gödel’s realism. It is the opinion of this author that dialogue with Gödel’s peculiar brand of realism offers the opportunity for insights and directives that simply cannot be gleaned from that of most other realists; that Gödel’s perfectionism--and perhaps even downright scrupulosity—with regard to his own work manifests a seriousness and attention to detail that is often wanting in the lackluster syncretism and lackadaisical pragmatism of many alternative realisms; that Gödel’s realism, to put it plainly (if lightly), deserves to be evaluated, analyzed, and critiqued—not merely as a stalking-horse for the critic’s own views, but for the sake of extracting the value and merit which can only be obtained (or at least is able to be attained most efficiently) through a serious and accurate analysis of that realism and the intellectual milieu within which it thrives.

To that end, this essay will pursue the following plan, which may be divided into two parts. The first part will begin by enumerating and describing the various elements that shape Gödel’s realism; then it will examine the arguments that Gödel puts forth directly to support that realism; from there, it will conclude with an evaluation of what this realism does and does not achieve. The second part will begin with a critique of both Gödel’s arguments and the system within which he presents them, and conclude with the author’s own remarks on the nature of Gödel’s contribution.

# 2 The nature of and arguments for Gödelian realism

## 2.1 Characteristics of Gödel’s realism

No philosophical thesis can be adequately understood without reference to the system of beliefs within which it thrives; just as knowing a person’s family history often sheds unexpected light on their personality, so too will coming to empathize with Gödel’s basic beliefs, concerns, and struggles bring with it an enhanced ability to fairly evaluate his brand of realism. The following remarks are meant to illuminate different important facets of the framework of Gödel’s realism.

First, it should be noted that by his own account, Gödel’s numerical realism is inextricably tied up with his conceptual realism—so much so, in fact, that Gödel’s arguments for the one often bleed into arguments for the other.[[1]](#footnote-1) Unlike Quine and others, it is certain that Gödel does not think that he can arrive at realism regarding mathematical objects without that realism being part of the larger package of conceptual realism.

But what, exactly, brought Gödel to place this realism concerning concepts at the center of his philosophy of mathematics? The answer to this question must be arrived at via an account of the philosophical figures to whom Gödel is reacting and by whom he is positively influenced.

It is widely acknowledged that one of the main targets of Gödel’s work in mathematics and philosophy is the finitist program of David Hilbert. In an unpublished paper likely written in the early part of the 1960’s, Gödel traces the genesis of the Hilbert program back to a generic wave of skepticism with regard to the objectivity of concepts, occasioned by the recent paradoxes of Cantor and Russell. Therefore, Hilbert seeks to avoid paradox by founding the certainty of all of mathematics on a limited, finite and “safe” base. As is old hat by now, however, Gödel’s incompleteness results shattered that dream. The lesson that Gödel gleans from this is that Hilbert is simply wrong to think axioms are definitions, and that in order to be valid, axioms must hold for any objects or concepts whatsoever. For Gödel, Frege is right to quip to Hilbert, “Given your definitions, I do not know how to decide the question whether my pocket-watch is a point” (TM 157). Hilbert may be right both to take the problems posed by paradox and the demands of logical rigor seriously, but he goes too far in barring conceptual analysis from the mathematical realm altogether. For Gödel, the process of deepening our mathematical knowledge “must thus consist, at least to a large extent, in a clarification of meaning that does not consist in giving definitions” (*\*1961/?*, 383).

A second and important aspect of Gödel’s reaction to Hilbert is the thesis that I shall refer to as “Gödel’s axiomatic infinitism” (abbreviated GAI). Although Gödel never states such a thesis explicitly, it forms the background of much of Gödel’s philosophy of mathematics, and can be gleaned easily from two different statements in Gödel’s work; the first statement being that “all of mathematics is reducible to abstract set theory” (*\*1951*, 305), and the second being that the axioms of set theory are infinite.[[2]](#footnote-2) This combination of claims leads to the logical conclusion that the foundations of mathematics are infinitely deep. Thus, Gödel’s reaction to Hilbert is linked up to his realism by two routes: firstly, by way of the fact that conceptual analysis cannot be replaced by forms of stipulation, which fact seems to inject mathematical concepts with an important degree of objectivity; secondly, via the thesis of GAI, which affirms the inexhaustibility of mathematics, and thereby ensures that Gödelian realism cannot be selective regarding the objective reality of different facets of mathematics in the way that, for instance, Quinean realism demurs from the upper echelons of set theory: if one accepts both the objectivity and inexhaustibility of meaningful mathematical concepts, then one’s mathematical realism must be wholesale.

The former half of this pair of beliefs, the claim that conceptual analysis cannot be ignored, is both explicit and thematic in Gödel’s critique of two other important figures to whom he reacts: Russell and Carnap. Against Russell, Gödel insists that concepts cannot be reduced to classes,[[3]](#footnote-3) nor can the latter be reduced to a string of type 0 objects.[[4]](#footnote-4) In order to affirm the objectivity of concepts in the face of the paradoxes, Gödel insists that the solution to the paradoxes is not the radical denial of all impredicative definitions (which is not entailed by the paradoxes, but only by the constructivist and reductionist attitude behind the no-class theory), but should be pursued via the “more conservative course” that “would consist in trying to make the meaning of the terms ‘class’ and ‘concept’ clearer, and [setting] up a consistent theory of classes and concepts as objectively existing entities” (*1944*, 152). In what is perhaps an encapsulation of the difference between his approach and Russell’s, Gödel sees the solution to the paradoxes in a more mature form of the Fregean conceptual realism that Russell saw as the underlying cause of the whole problem.

Against Carnap, Gödel goes a step further. Besides rejecting Carnap’s idea that a proposition is analytic if it can be reduced to a tautology,[[5]](#footnote-5) Gödel also sets up an altogether different criterion of analyticity: Gödel claims that a statement is analytic if it is “true owing to the nature of the concepts occurring therein” (*\*1951*, 321).

But in many respects, Gödel is not as far from Carnap as his critique would lead one to think. It may even be useful to see Gödel’s notion of analyticity not so much as a rejection of Carnap, but as a corrective and expansion of a basically Carnapian framework along Neo-Fregean lines.[[6]](#footnote-6) Such a view is bolstered when one considers two other aspects of Gödel’s philosophy of mathematics: his acceptance of the analytic/synthetic distinction and his pluralistic affinities.

From this first point, one can see that Gödel’s reaction to Carnap develops along completely different lines than that of Quine. Gödel, in fact, holds that having some form of an analytic/synthetic distinction marks a point of merit to Carnap’s system, even if Carnap’s own version of the analytic/synthetic distinction is skewed by the reductionism inherent to it. He writes:

It is correct that a mathematical proposition says nothing about the physical or psychical reality existing in space and time, because it is true already owing to the meaning of the terms occurring in it, irrespectively of the world of real things. What is wrong, however, is that the meaning of the terms (that is, the concepts they denote) is asserted to be something man-made and consisting merely in semantical conventions” (*\*1951*, 320).[[7]](#footnote-7)

From this one can see that Gödel’s realism, unlike Quine’s, brings with it the possibility of safeguarding one longstanding belief about mathematics: that our knowledge of it is *a priori*.

A second resemblance that Gödel bears to Carnap lies in his attraction to some form of mathematical pluralism. The main difference, however, is that while Carnap’s pluralism is a principle of tolerance towards a multiplicity of budding linguistic frameworks,[[8]](#footnote-8) Gödel’s pluralism is of a methodological sort. While Carnap’s commitment to the distinction between analytic and empirical truths is accompanied by a commitment to the verifiability criterion of meaning (which itself entails different methodologies for the different types of truths), Gödel is willing to blur the standard methodological lines between empirical and *a priori* sciences considerably. Thus Gödel argues that the relationship that mathematics bears toward the higher empirical sciences (such as physics) is neither that of a dictator to his subjects nor that of individuals disinterested in and unknown to each other; rather, the relationship between the two is one of reciprocal exchange and mutual benefit. Taking an assertive stance on this issue, Gödel writes,

If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics. The fact is that in mathematics we still have the same attitude today that in former times one had toward all of science, namely, we try to derive everything by cogent proofs from the definitions (that is, in ontological terminology, from the essences of things). *Perhaps* this method, if it claims monopoly, is as wrong in mathematics as it is in physics. (*\*1951*, 313)[[9]](#footnote-9)

As is evident from the above quote, Gödel’s methodological pluralism is itself closely linked to his realism; if mathematical objects are real and objective, then mathematical methodologies ought to reflect this, and ought not to be so detached from the rest of science.

One easily overlooked aspect of Gödel’s mathematical philosophy, but one which still needs to be mentioned, is his acceptance of classical mathematics. This is important because some of Gödel’s arguments for realism develop by way of creating a realist/intuitionist dichotomy, from whence the intuitionist restrictions on mathematics are dismissed as intolerable.

The final major constituent of Gödel’s mathematical realism, both in our analysis and in Gödel’s own chronology, is his appropriation of the phenomenological methods and insights of Edmund Husserl. Gödel began reading Husserl in 1959, and found that Husserl’s work managed to do justice both to Gödel’s own concerns and to the insights of Kantian idealism. At the heart of phenomenology, Gödel had found a philosophy that shared and gave weight to one of his own central tenets, namely, that an adequate philosophy of mathematics needs must pay attention to the unfolding of the conceptual content of mathematical propositions.

In sum, Gödel’s mathematical realism grows within and is nourished by an epistemic ecosystem containing the following beliefs: that there is a strong and real distinction between analytic and synthetic truths; that the foundations of mathematics in set theory are infinitely deep, because set theory is replete with an infinite of axioms; that realism concerning mathematical objects is unlikely to be justifiably attained without being accompanied by a realism concerning concepts; that every adequate philosophy of mathematics must pay due honor to descriptive conceptual analysis, which cannot be reduced to the stipulation of definitions; that a philosophy that *does* do justice to conceptual analysis may be found in the phenomenology of Husserl; that calls for the wholesale revision of classical mathematics are unwarranted and unnecessary; that because math is about the real world just as truly as zoology, the methods of mathematics ought to make room for methods that have proved their worth in the more abstract empirical sciences. These are the main tenets of Gödel’s philosophy of mathematics.

## 2.2 Gödel’s Arguments for Realism

From here, let us embark on an analysis of the arguments that Gödel gives in support of his realism. I shall examine four, all of which excluding the first I have taken from the Gibbs lecture of 1951.

The first argument that Gödel puts forth in support of his realism comes as a response to Russell in his 1944 paper “Russell’s mathematical logic”. There, after unpacking the ambiguity of Russell’s vicious circle principle, Gödel argues that Russell’s ban on impredicative definitions only holds for one version of that principle (i.e. the version stating that no entity can be *defined* exclusively in terms of a collection to which it would antecedently belong), and even then the principle only holds if one insists on taking a constructivist viewpoint regarding the entities of mathematics. But if one takes the objects of mathematics to exist objectively and “independently of our constructions, [then] there is nothing in the least absurd in the existence of totalities containing members which can be described…only by reference to the totality” (*1944*, 136). Therefore, the need for a ban on impredicative definitions is not as straightforward as Russell himself presents it as being. Thus Gödel poses a dilemma: if one wishes to hold on to the patrimony of classical mathematics, one must be a realist with respect to mathematical objects; if one takes the projectivist/constructivist viewpoint, however, one must, having been divested of significant portions of classical mathematics, be prepared to be satisfied by an attenuated and half-starved quasi-intuitionistic mathematics.

Gödel’s second argument is a complicated and sophisticated dichotomy based on the first incompleteness theorem and directed against Carnap’s philosophy of mathematics. Gödel argues that since a Carnapian framework is itself a well-defined system that can be formalized and axiomatized, any Carnapian framework that contains a certain amount of arithmetic, if it is sound, will be incomplete. Furthermore, such a framework will contain a formal proof of its consistency if and only if it is inconsistent. The resulting dilemma is as follows: either there are absolutely unknowable mathematical objects and facts that “exist objectively and independently of our mental acts and decisions”, or there are methods of proof that are beyond the scope of finite formal/mechanistic systems; that is, the human mind “infinitely surpasses the powers of any finite machine” (*\*1951*, 311, 310) (Gödel himself thinks that both horns of this dilemma are in fact true). If one chooses the former half of this dilemma, then one must be a truth value realist; if one chooses the latter, then, apart from the entailment of a commitment to some form of vitalism, one seems to be constrained to confess the necessity and usefulness of descriptive conceptual analysis for mathematics, the force of which only seems to make sense if one assumes some form of realism.

Gödel’s next argument is similar, but is aimed more specifically at proving the objective existence of concepts. In a critique that he shares with Quine, Gödel argues that no Carnapian framework is able to formulate a comprehensive list of those truths which are analytic by virtue of their meaning in a way that is not ultimately question begging. But again here, Gödel explicitly ties his critique to his incompleteness results, this time to the second theorem. Gödel argues that “a proof for the tautological character…of the axioms is at the same time a proof for their consistency, and cannot be achieved with any weaker means of proof than are contained in these axioms themselves” (*\*1951*, 317). To be more specific, a Carnapian framework, without drawing on richer resources than those contained in it, can only prove the analyticity of the statements contained in it if it is, in fact, inconsistent. Therefore, if one expects to get beyond mere stipulation, one must admit that there are relations that exist between concepts that are non-stipulative. But if mathematical concepts are our creations, then it makes little sense for them to have the sort of rigid constructive constraints that they do. Therefore, mathematical objects and truths (or at least something in them) exist objectively.

The last argument that Gödel puts forth in *\*1951* is more informal, and bears a similarity to the Quine-Putnam indispensability argument. Gödel argues that, “there exists no rational justification of our precritical beliefs concerning the applicability and consistency of classical mathematics . . . on the basis of a syntactical interpretation” (318). Although Gödel knows that such a consideration is not decisive for realism, inference to the best explanation should lead one to the belief that acceptance of realism in truth value and ontology is the most reasonable option.

One final point of note indirectly pertinent to Gödel’s realism is a brief remark that he makes regarding psychologism. On this, Gödel writes the following:

The chief objection to this view I can see at the present moment is that if it were correct, we would have no mathematical knowledge whatsoever. We would not know, for example, that 2+2=4, but only that our mind is so constituted as to hold this to be true, and there would then be no reason whatsoever why, by some other train of thought, we should not arrive at the opposite conclusion with the same degree of certainty. (*\*1951*, 322)

The inference that one may draw from such a situation is that if neither nominalism nor psychologism is in the last analysis a reasonable option, one seems constrained to accept some form of realism, be it of a Platonic or an Aristotelian form.

## 2.3 Concluding Remarks on part 1.

But what, exactly, do these arguments and considerations achieve? Do they make realism a foregone conclusion? Overall, I think that Gödel’s own self-assessment is both modest and accurate. At the conclusion of the Gibbs lecture, he states:

I do not claim that the foregoing considerations amount to a real proof of this view about the nature of mathematics. The most I could assert would be to have disproved the nominalistic view, which considers mathematics to consist solely in syntactical conventions and their consequences. There are, however, other alternatives to Platonism, in particular psychologism and Aristotelian realism. In order to establish Platonistic realism, these theories would have to be disproved one after the other, and then it would have to be shown that they exhaust all possibilities. (*\*1951*, 322)

But even if these were the only results, a refutation of nominalism in all of its forms would be a striking achievement; and I think that in this, Gödel’s success is both stunning and definitive. Of course, my judgment on this point is bound to be controverted by many; but it is unclear how any nominalist objection to Gödel’s argument could avoid degenerating into a semblance of that of Achilles’ famous testudinate interlocutor. The main thrust of Gödel’s most forceful anti-nominalist arguments rest on the following thoroughly benign presumptions: 1) that the incompleteness results can be reproduced in any formal system with a certain amount of arithmetic; 2) that a proof of the tautological and/or logical character of all of the analytic truths of a framework is fundamentally a consistency proof for that framework, which that framework on its own can achieve only if it is inconsistent. If the nominalist wants to avoid the conclusions that Gödel draws from the incompleteness results, he must either a) set up some *ad hoc* axioms to block the incompleteness proofs for his system (in which case the nominalist still lacks any *reason* for preferring such a system to another that lacks this provision) or b) simply insist by sheer act of will on holding to a given incomplete and/or inconsistent framework or combination of frameworks, which decision would amount to nothing less than the abandonment of the laws of logic and/or the quest for truth itself. The nominalist certainly *has* this option, but to take it is to *de facto* empty oneself of all intellectual credibility.

So with the nominalistic option shipwrecked, the situation is exactly as Gödel envisions it: of those options that have explicitly been enumerated up to the present point in history, only psychologism and varieties of realism remain.

Also, though his direct comments on it are brief, I think that Gödel’s thoughts on psychologism are accurate, namely, that it is impossible to take a psychologistic approach to mathematical objects and simultaneously hold that there is any such thing as mathematical truth *simpliciter*. If one chooses the psychologistic route, then one cannot be a truth-value realist regarding mathematical propositions.

In conclusion, Gödel’s arguments are sufficient for a refutation of the types of nominalism represented both by Carnap and by formalist mathematics. Furthermore, they lay bare the case that, against psychologism, only a realist ontology can secure truth value realism.

# 3 Critique of Gödelian Realism

## 3.1 On Gödel’s arguments

In this section I intend to critique the arguments that Gödel sets forth in favor of his particular form of realism.

The first argument with which I will deal is that which I have above referred to as being in the family of the indispensability arguments. This argument is often considered to be a fairly strong argument in favor of ontological realism in the realm of mathematics, and Hartry Field even goes so far as to call it the “one and only serious argument for the existence of mathematical entities (1980, 5). But I must confess that I find it to be, in fact, one of the *weakest* arguments for realism, for the simple reason that the same argument seems to have been as applicable to Newtonian absolute space and Ptolemaic epicycles in past times as it is to numbers today. If this is the strongest justification for a realist ontology regarding numbers, then the argument by way of the so-called pessimistic induction (i.e. the idea that since all of our best past scientific theories regarding the necessary existence of certain ontological entities have been proven false, it is unlikely that our current theories, including those that support the ontological existence of numbers, are true either) is likely enough to sufficiently fell such a case. This is why Field’s project comes across not as a foolish enterprise, but instead is viewed as a great philosophical endeavor and an achievement of serious merit. This is likely also why Gödel does not rest his case on this argument, but merely mentions it in passing as an additional consideration in favor of his position. Gödel seems to understand that considerations of applicability may be an auxiliary weight that bear on our ultimate decision to adopt a realist standpoint, but that such considerations cannot provide a sufficient foundation for realism in mathematics.

I have one comment regarding an ambiguity in Gödel’s argument for realism from the use of impredicative definitions. When Gödel argues that someone who takes a constructivist approach regarding mathematics must give up a significant portion of classical number theory, it seems that Gödel is using the word ‘constructivism’ specifically to refer to a certain type of stipulative constructivism that one can most readily associate with those aspects of philosophy of mathematics broadly shared by Frege, Russell and Carnap. This is most important because while the most prominent branch of psychologism—intuitionism—is certainly a form of constructivism, and *does* reject significant portions of classical logic and mathematics, it is unclear whether Gödel means to suggest that *every* type of psychologism must accept the intuitionist restrictions. In particular, it does not seem to be the case that Gödel means to suggest that every neo-Kantian or psychologist interpretation of mathematics must be intuitionistic in the restricted sense of the term; and if Gödel did mean to include this latter judgment in his argument against Russell (which seems unlikely given the neo-Kantian elements in Gödel’s own thought), he did not sufficiently make his case. Lastly, given that Gödel’s dichotomy does not explicitly address the person who chooses intuitionism over realism in the posed dilemma other than by briefly pointing out the negative consequences of intuitionism for both mathematical and scientific practice (although, given the context and purpose of Gödel’s essay, he cannot be faulted not pursuing this line of argument further), Gödel’s argument against taking the intuitionistic option is reducible to another variant of the indispensability argument, which, as pointed out above, is not particularly compelling.

## 3.2 Digression on Husserl’s phenomenology

Before returning to consider Gödel’s arguments based on incompleteness, I would like to make some comments on Husserlian phenomenology and Gödel’s appropriation of it. In one of his later works, giving an emphatic endorsement of the utility of Husserl’s work for philosophy of mathematics, Gödel refers to Husserl’s phenomenology as “the beginning of a science which claims to possess a systematic method for. . . . a clarification of meaning” and states, “I believe there is no reason at all to reject such a procedure at the outset as hopeless” (*\*1961/?*, 383). While such comments may come across as defensive in posture and relatively modest, this has less to do with Gödel’s assessment of Husserl (which was highly positive) and more to do with his pessimistic view of the state of philosophy of mathematics in his day (Gödel regarded much of it as dogmatically anti-realist and positivist).

But if Husserl’s phenomenology played such a seminal role in Gödel’s mature thought, it behooves us to know something both about it and about the way that it manifests itself in Gödel’s own work.

As stated earlier, Gödel’s main interest in phenomenology stems from his belief that any adequate account of mathematics must be open to the use of procedures that consist “in a clarification of meaning that does not consist in giving definitions” (ibid). Husserl’s phenomenology attempts to do this. But the method by which Husserl attempts to do this is not as straightforward as one might think. According to Gödel,

Clarification of meaning consists in focusing more sharply on the concepts concerned by directing our attention in a certain way, namely, onto our own *acts* in the use of these concepts, onto our powers in carrying out our acts, etc. (ibid)

The emphasis on acts here is important. According to Husserl, our understanding of the concept of number is bound up with introspection of our actions, because numbers themselves are “mental creations insofar as they are the results of activities which bear on concrete contents (Husserl 106). Thus, there is a strong sense in which Husserl is indebted to Kant, and Gödel through Husserl on this point. Husserl is very emphatic on this point[[10]](#footnote-10), and it is precisely in connection with this relation between numbers and psychic activity that Husserl seeks to rest two points: first, that numbers are acausal and non-spatiotemporal objects; second, that they are nevertheless accessible to human thought. As one prominent scholar on Gödel’s use of phenomenology writes,

A central reason to avoid earlier, naïve forms of Platonism is that they place essences outside of all possible experience. They treat essences as abstract things-in-themselves. On the phenomenological view, however, we are clearly directed toward and have access to essences. (Tieszen 1998, 200)

Now at this point two questions arise, the first being whether such a position does not compromise Gödel’s realism, and the second being “what, then, distinguishes Gödel’s realism from intuitionism?”

The latter reservation may be answered fairly quickly by referring the reader back to Gödel’s methodological pluralism and his acceptance of classical mathematics. Unlike Brouwer and other intuitionists, Gödel does not seek to build mathematics *exclusively* on a foundation of Kantian intuition. The fact that Gödel is not an intuitionist does not in any way bar him from adopting Kantian insights in support of and alongside his own views, and this is in a certain sense precisely what Gödel is doing via his appropriation of Husserl.

A further answer to the same that also addresses the first question is based on the Husserlian idea of *intentionality*. As stated prior, numbers concepts are, according to Husserl, developed by reflection on human psychical acts. But such acts are always *directed*. As Tieszen writes, “This means that cognition is always about something. Consciousness is always consciousness *of*” (ibid 182). In other words, our concepts are not purely subjective. Because they are about something, our concepts must clearly be linked to the noumenal realm. But precisely because things like numbers are concepts and not concrete entities, one cannot neglect or ignore the mind’s subjective contribution when considering their ontological nature. With respect to this point, Gödel, who earlier wrote that “mathematical objects and facts (or at least something in them) exist objectively and independently of our mental acts and decisions” (*\*1951*, 311), seems to have later moved closer to the latter option through the influence of Husserl. Hence Gödel later claims that

Mathematical intuition need not be conceived of as a faculty giving an *immediate knowledge* of the objects concerned. Rather it seems that…we *form* our ideas also of these objects on the basis of something else which *is* immediately given. (*1964*,271)

In Husserl, Gödel comes to develop an appreciation for what he holds to be the kernel of truth in Kantian idealism, and Gödel sees in Husserl a *rapprochement* between idealism and naïve realism that overcomes the weaknesses of both.

One final difference between Gödel’s view and intuitionism is that while, for an intuitionist, the only mathematical entities and formulae that are to be admitted are completed or fulfilled intentions, Gödel’s criterion is looser. For Gödel, a concept need not be *completely* clear and intelligible in order to be admitted into mathematical use, just so long as it is *sufficiently* so. This, for instance, explains the difference between their attitudes towards completed infinites. It also explains why Gödel is convinced that there must be a clear-cut answer to Cantor’s continuum problem.

On a different note, an important distinction between Gödel and Husserl lies in their respective attitudes toward science. As stated above, Gödel’s methodological pluralism leads him to a positive assessment of the state and methods of modern science. Gödel is even committed to the view that enumerative induction deserves a place in mathematical methodology.[[11]](#footnote-11) Gödel’s positive assessment of science is also important in his defense against intuitionism.

Husserl, on the other hand, is sharply critical of scientific methodology within his day, claiming that “All natural science is naïve with regard to its point of departure” (Husserl 1965, 171). As such, Husserl both calls for a reassessment of the aims, capabilities, and legitimate achievements of science, and is especially antipathetic toward the view that mathematics ought to—or even can—adopt methods peculiar to empirical science. For Husserl, “The relationships in the sphere of the psychical are totally different from those in the physical sphere” (ibid, 179). Husserl thinks of the physical sphere in much the same way as we do today: it is the realm of rocks, trees, coffee cups, porpoises and perhaps electrons. The psychical sphere, by contrast, finds its origin in human psychical acts: it is the realm of the *a priori*, and is the source not only of geometry and number theory, but also of all tradition, of which geometry is a particular example. The sum total content of the traditions of a particular group (in our case, 21st century America, and more broadly, Western Civilization) is what is signified by the term "*culture*". Because of such a sharp dichotomy between empirical being and this latter realm, what Husserl calls the *Leibwelt*, or Lifeworld, there necessarily exists an insurmountable chasm between the methods and aims appropriate to each.

## 3.3 Recommencement of Critique

Now let us return to our critique. It should be clear by now that much of Gödel’s realism can be traced back to two primary supports: the first consisting in his insistence on the importance of conceptual analysis, his conceptual realism, and his appropriation of phenomenology; the second related to his own incompleteness theorems. But though these are separate, they should not be divided too sharply: for instance, Gödel often uses his incompleteness theorems to make a case for the necessity of descriptive conceptual analysis.

Here I would like to pose two sets of questions: 1) How did Gödel regard the incompleteness results? How does Gödel view the relationship between incompleteness and phenomenology? 2) How should we regard those results? What is the proper relationship between incompleteness and phenomenology?

Regarding the first set, it is clear that Gödel viewed his incompleteness results as a support for his conceptual realism, and Gödel even credits his realism as a major motivation underlying his achievement. Thus, the incompleteness results are regarded by Gödel as having real positive consequences even if those consequences are not conclusive. And phenomenology is just the fulfillment of and more perfect form of that same conceptual realism that is pointed to by the incompleteness results. Therefore, the relationship between them is one of *linear progression*.

Gödel also thinks that the setting up of new formal systems is able to be the occasion for insights into new concepts, which themselves will entail the further development of formal systems. Therefore, the incompleteness results should spur one on to the development of more complete systems. Gödel writes,

The certainty of mathematics is to be secured…by cultivating (deepening) knowledge of the abstract concepts themselves which lead to the setting up of these mechanical systems, and further by seeking *according to the same procedures*, to gain insights into the solvability and the actual methods for the solution of all meaningful mathematical problems (*\*1961/?*,383).

Therefore the relationship between phenomenology and incompleteness is also one of *mutual support*.

Now at this point, one could formulate the following position: Mathematical entities are concepts; concepts are directed intentions; intentions are psychic acts: therefore, if one merely combines a Davidsonian event semantics with a Quine’s criterion of existence, realism in the ontology of mathematics necessarily follows. But I would like to suggest that such a solution is not so easily obtainable for several reasons.

First, it is in no way clear that mathematical entities are, in fact, reducible to intentional psychic activity. Gödel nowhere argues for this, and while I cannot go into detail on this point here, insofar as Husserl’s own arguments for the position often presuppose both a Cartesian dualism regarding the relationship between the world of things and the world of culture, as well as a Kantian interpretation of the categories as categories *of thought*, I find them to be fairly weak and tendentious.

Furthermore, if one *does* accept a Husserlian analysis of the nature of mathematical entities and combines it with a belief in the objective reality of concepts, one is faced with a frightening ontology of Meinongian magnitude. In other words, if mathematical objects can be said to exist simply because they are formed on the basis of some given, so too can golden mountains, invisible fat men in doorways, Pegasus, Peter Pan’s evil twin, and the T-rex that just bit off my left leg. An ontological criterion with such gruesome results cannot be swallowed so easily.

Third a Husserlian mathematical ontology is simply too close to intuitionism for it to clearly be a form of realism. Richard Tieszen points out a historical link between the two, writing that

Heyting and Becker, for example, established an interesting connection with Husserl’s work by identifying mathematical constructions (in the sense of constructive mathematics) with fulfilled mathematical intentions, where the notion of ‘intention’ is understood in terms of the theory of intentionality. (Tieszen 1998, 182)

Now given both the historical connection and the doctrinal similarities between intuitionism and Gödel’s Husserlian ontology, it is unclear how a debate between Gödelian realism and intuitionism does not merely degenerate into a semantic dispute. Between the two, the main distinction is simply a question of when a concept is sufficiently clear as to be admitted into general mathematical use—a distinction of degree and not of kind; and furthermore, this distinction is not necessarily relevant to the ontological debate: both equate numbers with intentional concepts, the one party laying emphasis on the subjective contribution, the other on the objective restrictions. But neither in principle objects to the basic analysis of the other party. Therefore, before calling Gödel’s story about the ontology of mathematics a realism, one must (apart from having a clear and meaningful criterion for what realism is—itself a question yet to be satisfactorily answered) either explain the precise extent and significance of Gödel’s departure from intuitionism or one must show that intuitionists are realists after all.

Lastly, and most importantly on this point, *Husserl himself does not hold that conceptual analysis of an object entails its existence, and Gödel himself nowhere argues for this connection*. Husserl writes, “Pure phenomenology as science, so long as it is pure and makes no use of the existential positing of nature, can only be essence investigation, and not at all an investigation of ‘being-there’” (Husserl 1965, 183). In other words, essence investigation need not entail existence, and Husserlian phenomenology is not concerned with the positing of such. Without Gödel providing an independent argument for the contrary, it seems to be the case that Gödel is merely tacking his existential claims onto Husserlian phenomenology without offering any justification for doing so.

But perhaps unbeknownst to Gödel, a realism of the Husserlian type that insists that conceptual analysis be taken seriously offers perhaps the best grounds upon which one could object to the incompleteness results as having anything but negative consequences. To put it plainly, in any system that was not of the Hilbert/Carnap stipulative type, one would *not* be free to merely stipulate that there could be a sentence that was simultaneously true and unprovable: one would first have to resort to a rigorous conceptual analysis of the relationship between provability and truth. And prior to working out the details of said relationship, any system of mathematics that *did* attempt to reconcile the partition between conceptual analysis and formal systems would have clear reasons for having reservations about the introduction of the Gödel sentence. In a more free-flowing system such as this, the incompleteness results could not have positive implications for an ontology for the simple reason that they would not be able to get off the ground.

…Unless, of course, one embraces Gödel’s methodological duality in mathematics that seeks to put the considerations of fruitfulness in physics on par with more traditional methods of analysis. But here Husserl’s point is especially apposite. Husserl states that entities of the realm of culture are so different from those of the empirical world that to admit such methods as induction into the former realm would be absurd. Regarding geometry specifically (though the same point could in principle be made about arithmetic and the other mathematical sciences) he writes,

“What we know—namely, that the presently vital cultural configuration ‘geometry’ is a tradition and is still being handed down—is not knowledge concerning an external causality which effects the succession of historical configurations as if it were knowledge based on induction, the presupposition of which would amount to an absurdity here. (1970, 264-65)

Given the plausibility of the above claim, it is still unclear how such permissiveness in methodology is not likely to lead to a thorough breakdown of the *a priori/a posteriori* distinction; it is unclear how this methodological dualism would fail to do anything but move Gödelian realism closer to Quinean realism, and it is unclear how Gödel’s methodological pluralism does not compromise both his commitment to the *a priori* character of mathematical knowledge and to the non-causal, non-spatiotemporal character of mathematical entities. But to the degree that mathematics really *does* embrace the criterion of fruitfulness and the peculiar form of instrumentalism that comes with it, the argument for the existence of numbers becomes identical in form to the argument for the existence of, say, epicycles or electrons—that is, such an argument transforms itself into an instance of the indispensability argument, which, as stated earlier, is more easily dispensed with than its name implies.

Therefore I conclude that, since every attempt to squeeze positive content out of them leads to undesirable and sometimes downright bizarre consequences, the incompleteness results ought to be interpreted strictly as a *reductio*, and can only be regarded as having a negative significance. Therefore, one can neither build nor support a realist ontology on those results.

# 4 Conclusion

One may be inclined to ask, however, “What exactly are these arguments meant to show? Has it not already been explicitly stated by Gödel himself that he has not brought forth arguments sufficient to prove his own realist thesis so much as he has attempted to refute a very specific strand of anti-realist thought? Is one not, here, faulting Gödel for not achieving a goal which he did not so much as claim to have attempted?”

While this point may have a semblance of validity to it on the surface, it fails to understand that the argument above goes deeper than claiming that Gödel failed to bring forth sufficient arguments in favor of a realist ontology: the above attempts to show that Gödel’s particular brand of syncretism *cannot* serve as a foundation for a realist mathematical ontology, because it is internally inconsistent. Insofar as a Gödelian realist may emphasize the phenomenological elements of said realism, he either drifts towards intuitionism and transforms the ontological debate between those two camps into a semantic debate, adheres to Husserl’s warning about avoiding existential posits in the phenomenological realm and fails to secure realism, or ignores Husserl’s position and ends up letting a host of unruly entities into the ontological framework; insofar as a Gödelian realist may emphasize the empirical elements of Gödel’s thought, Gödelian realism collapses into a Quinean realism where the case for realism is reducible to the unconvincing and unpersuasive indispensability argument, and either loses the *a priori*/*a posteriori* distinction so central to Gödel’s thought or raises anew age-old questions about the applicability of mathematics. In short, Gödel shows no proof that he can harmonize the various aspects of his realism, and there are good reasons for believing that he cannot and that even if he could, neither of the chief polarities of his realism—Husserlian phenomenology and the criterion of fruitfulness—are adequate to Gödel’s ontological aims.

In full disclosure, I am inclined to think both that Gödel’s arguments against nominalism and psychologism are sound and that Gödel’s realist intuitions are basically correct. But the various positive elements are neither strongly developed in themselves nor well-harmonized with each other. Therefore, in spite of the value of Gödel’s philosophy of mathematics (which ought not to be underestimated), the realist in mathematical ontology must look elsewhere for a firm argumentative grounding for his/her convictions.

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1. e.g. “For someone who considers mathematical objects to exist independently of our constructions and who requires only that the general mathematical concepts must be sufficiently clear for us to be able to recognize their soundness and the truth of the axioms concerning them, there exists, I believe, a satisfactory foundation for Cantor’s set theory” (*1964*, 262). [↑](#footnote-ref-1)
2. “So the problem at stake is that of axiomatizing set theory. Now if one attacks this problem, the result is quite different from what one would have expected. Instead of ending up with a finite number of axioms, as in geometry, one is faced with an infinite series of axioms, which can be extended further and further, without any end being visible and, apparently without any possibility of comprising all these axioms in a finite rule producing them” (*\*1951*, 306) [↑](#footnote-ref-2)
3. “Classes and concepts may, however, also be conceived as real objects, namely classes as ‘pluralities of things’ or as structures consisting of a plurality of things and concepts as the properties and relations of things existing independently of our definitions and constructions” (*1944*, 128). [↑](#footnote-ref-3)
4. “One may, on good grounds, deny that reference to a totality necessarily implies reference to all single elements in it or, in other words, that ‘all’ means the same as an infinite logical conjunction” (*1944*, 135-136). [↑](#footnote-ref-4)
5. “[The Carnapian view holds that] mathematical propositions are true solely owing to the definitions of the terms occurring in them, that is, that by successively replacing all terms by their *definientia*, any theorem can be reduced to an explicit tautology, a=a” (*\*1951*, 315). [↑](#footnote-ref-5)
6. Gödel’s idea of analyticity is somewhat reminiscent of the following passage from Frege’s *Grundlagen*: “When…a proposition is called a posteriori or analytic in my sense, . . . it is a judgment about the ultimate ground upon which rests the justification for holding it to be true . . . The problem becomes . . . that of finding the proof of the proposition, and of following it up right back to the primitive truths. . . If, in carrying out this process, we come only on general logical laws and on definitions, then the truth is an analytic one. . . If, however, it is impossible to give the proof without making use of truths which are not of a general logical nature, but belong to the sphere of some general science, then the proposition is a synthetic one” (Frege 1884: §3). The main difference between Gödel’s view and Frege’s is that Gödel has a marked distinction between definition and a more descriptive approach to conceptual analysis, while Frege does not. [↑](#footnote-ref-6)
7. See also the remark at *\*1953/9*: “The syntactical point of view as to the nature of mathematics doubtless has the merit of having pointed out the fundamental difference between mathematical and empirical truth. This difference, I think rightly, is placed in the fact that mathematical propositions, as opposed to empirical ones, are true in virtue of the concepts occurring in them” (356-57). [↑](#footnote-ref-7)
8. “Let us grant to those who work in any special field of investigation the freedom to use any form of expression which seems useful to them; the work in the field will sooner or later lead to the elimination of those forms which have no useful function” (Carnap 1950, §5) [↑](#footnote-ref-8)
9. This breakdown of methodological barriers is advocated by Gödel throughout the whole of his philosophical career. Hence, Gödel presents this view in *1944*, when he writes, “the axioms of logic and mathematics. . . need not necessarily be evident in themselves, but rather their justification lies (exactly as in physics) in the fact that they make it possible for these ‘sense perceptions’ to be deduced” (127), and again in *1964*, stating that “Besides mathematical intuition, there exists another criterion of the truth of mathematical axioms, namely their fruitfulness in mathematics and, one may add, possibly also in physics” (272). [↑](#footnote-ref-9)
10. Regarding concepts in general, of which numbers are a specific instance, Husserl writes, ““The contents are [in the case of psychical relations], unified precisely by the act alone; and the unification, therefore, can only be noticed by means of a special reflection upon the act” (Husserl 1972, 113). [↑](#footnote-ref-10)
11. “If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics” (*\*1951*,313) [↑](#footnote-ref-11)